ST PANCRAS

St. Pancras Catholic Primary School

Mathematics Calculation Policy

Introduction

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (National Curriculum 2014)

This calculation policy has been largely adapted from the White Rose Maths Hub Calculation Policy and has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on how appropriate calculation methods are represented to the children and how these representations are developed to ensure continuity and progression throughout school. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. When considering the concrete, pictorial and abstract, it is important to state that they are not restricted to each year group. Children should receive exposure to all three and should feel that they can select the appropriate method. Similarly, as progressing through the school, children will be encouraged to use the pictorial and concrete approach to prove their understanding of the abstract calculation.

EYFS & Milestones

The EYFS Statutory Framework 2014 sets standards for the learning, development and care of children from birth to five years old and supports an integrated approach to early learning. This is supported by the 'Development matters' non-statutory guidance. The EYFS Framework in relation to mathematics aims for our pupils to:

- develop and improve their skills in counting
- understand and use numbers
- calculate simple addition and subtraction problems
- describe shapes, spaces, and measures

Concrete, Pictorial and Abstract

The content is set out in progression blocks (concrete, pictorial and abstract) under the following headings: addition, subtraction, multiplication and division. It is critical that children are taught maths in a way that develops a deep conceptual understanding as this is the only way of securing solid foundations. One way in which we can help children to do this is by using the concrete-pictorial-abstract (CPA) approach.

Using the concrete-pictorial-abstract approach: Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation

In this stage a child is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating balls into three equal groups or by sharing 10 biscuits among 5 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation

In this stage a child has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise, this could be the action of circling objects.

Abstract representation

In this symbolic stage a child is now capable of representing problems by using mathematical notation, for example: 10 ÷ 2 = 5. Children only use abstract numbers, figures and symbols (+-x÷) when they have enough context to understand what they mean. This is the 'final' and most challenging of the three stages.

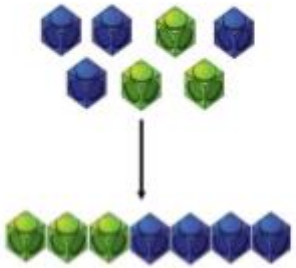
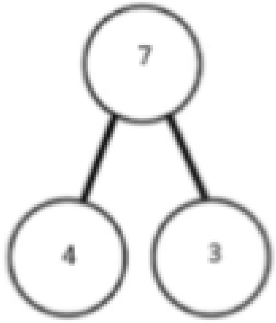
Teaching & Learning

It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the **focus must always remain on breadth and depth** rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems.

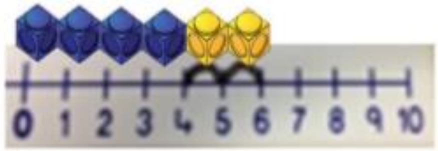
Teachers will use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept and develop themselves as mathematicians.

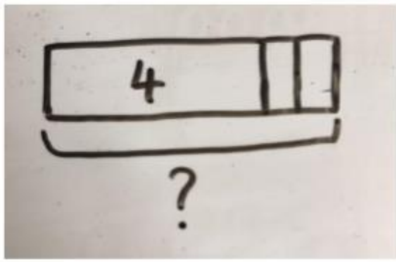
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as'.

<table>
<tr><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).

3+4=7</td>
<td>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.


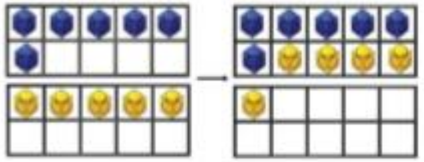
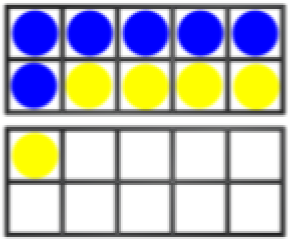
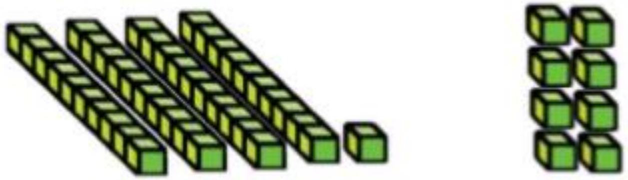
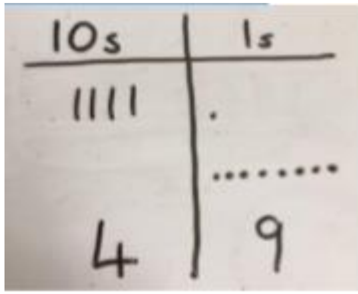
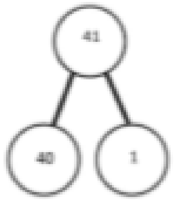
3+4=7</td>
<td>4 + 3 = 7
Four is a part, 3 is a part and the whole is seven.

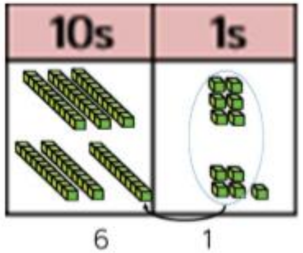
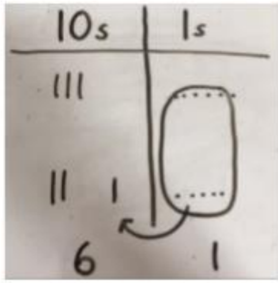
3+4=7</td>
</tr>
<tr>
<td>Counting on using number lines using cubes.


4+2=6</td>
<td>A bar model which encourages the children to count on, rather than count all.


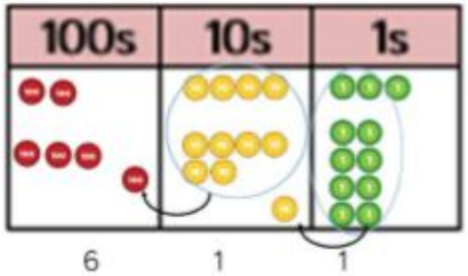
4+2=6</td>
<td>The abstract number line:
What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2? 4 + 2


4+2=6</td>
</tr>
</table>

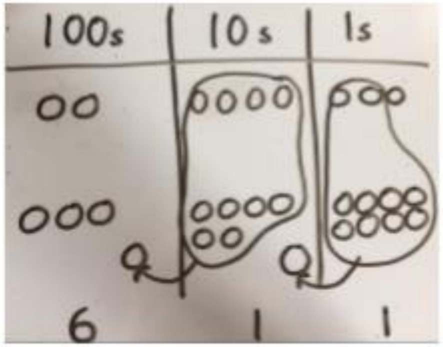
<table>
<tr>
<td>Regrouping to make 10; using ten frames and counters/cubes or Numicon.
</td>
<td>Children to draw the ten frame and counters/cubes.
</td>
<td>Children to develop an understanding of equality e.g.
$6 + \square = 11$
$6 + 5 = 5 + \square$
$6 + 5 = \square + 4$</td>
</tr>
<tr>
<td>TO + O using base 10. Continue to develop understanding of partitioning and place value.
41 + 8
</td>
<td>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.

</td>
<td>Flexible splits
41 + 8


$1 + 8 = 9$
$40 + 9 = 49$
$\begin{array}{r} 41 \\ +\ \ 8 \\ \hline 49 \end{array}$</td>
</tr>
<tr>
<td>TO + TO using base 10. Continue to develop understanding of partitioning and place value.
36 + 25

</td>
<td>Children to represent the base 10 in a place value chart. This below is known as regrouping.

</td>
<td>Looking for ways to make 10.
36 + 25=
1 5
$30 + 20 = 50$
$5 + 5 = 10$
$50 + 10 + 1 = 61$
Formal method:
$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$</td>
</tr>
</table>

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



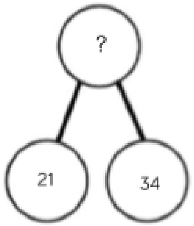
Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline {\scriptstyle 1\ 1\ } \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34

Part whole model



?	
21	34

Word Problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

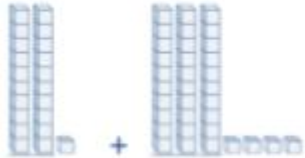
21 + 34 =

$\boxed{}$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

Formal written method

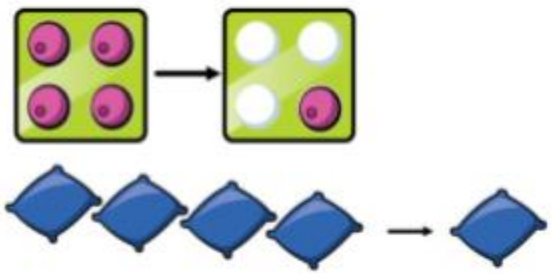
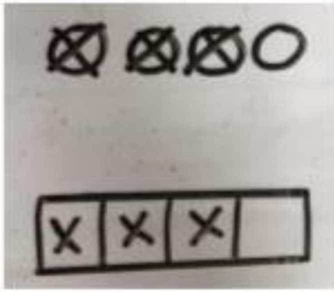
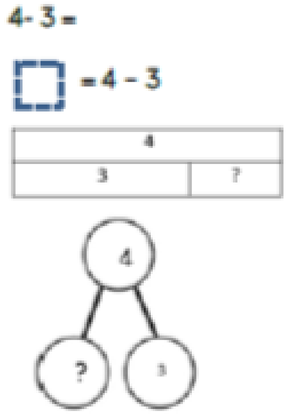
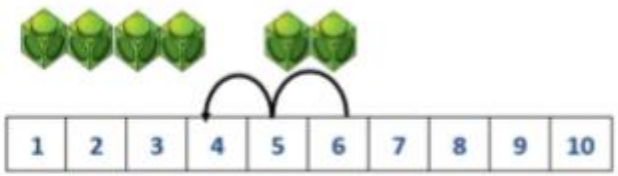
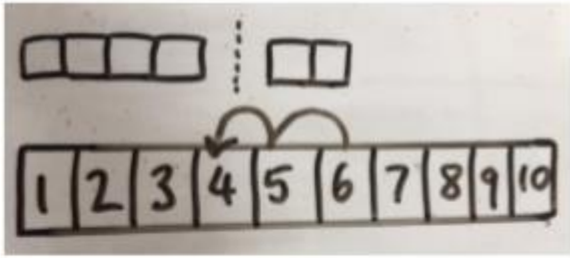
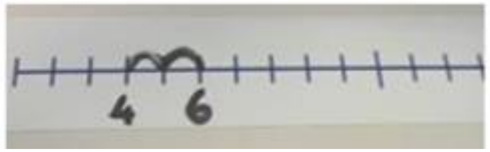
Problem solving



Missing digit problems:

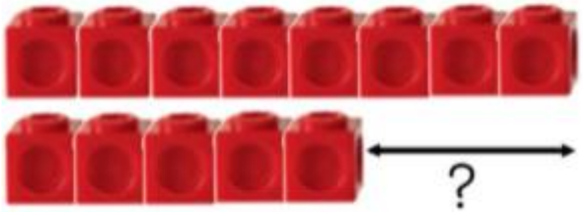
Calculation policy: Subtraction

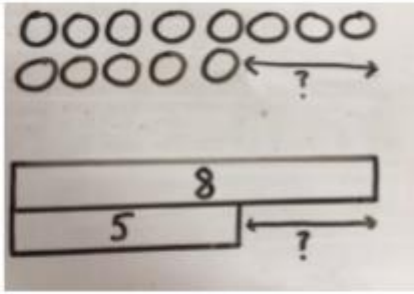
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).  4-3=1	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.  4-3=1	 4-3=1
Counting back (using number lines or number tracks) children start with 6 and count back 2.  6-2=4	Children to represent what they see pictorially e.g.  6-2=4	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.   6-2=4

<table>
<tr>
<td>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

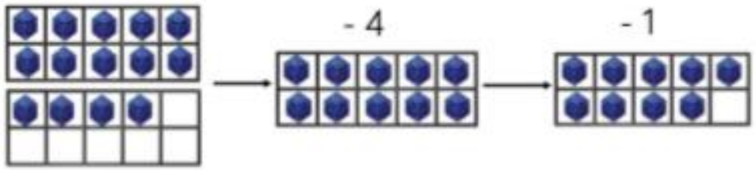
Calculate the difference between 8 and 5.

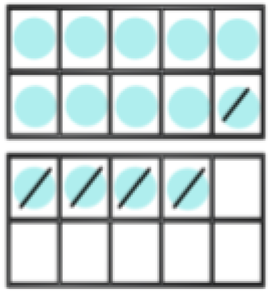

</td>
<td>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.


</td>
<td>Find the difference between 8 and 5.

8 − 5, the difference is $\boxed{}$

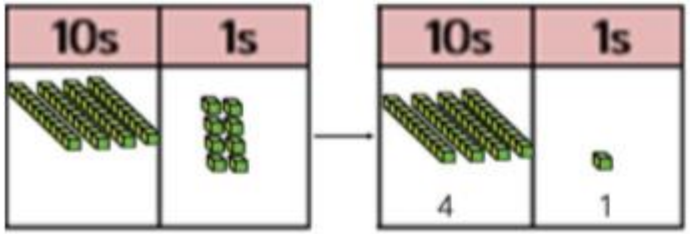
Children to explore why
9 - 6 = 8 − 5 = 7 − 4 have the same difference.</td>
</tr>
<tr>
<td>Making 10 using ten frames.

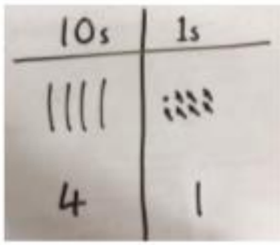


14-5=9</td>
<td>Children to present the ten frame pictorially and discuss what they did to make 10.

14-5=9</td>
<td>Children to show how they can make 10 by partitioning the subtrahend.

14 - 5 = 9
4 1

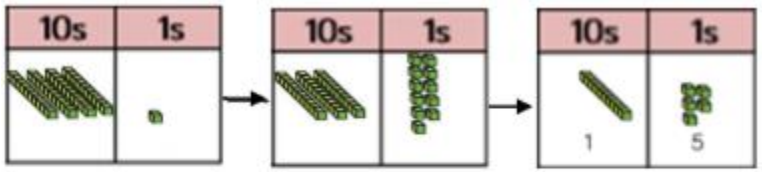
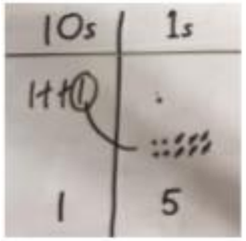
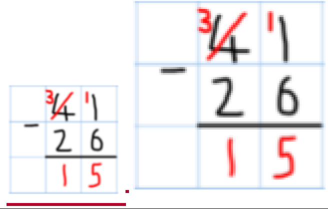
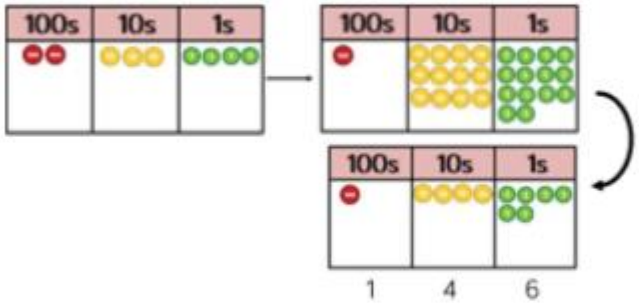
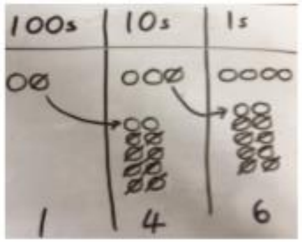
14 − 4 = 10
10 − 1 = 9</td>
</tr>
<tr>
<td>Column method using base 10.



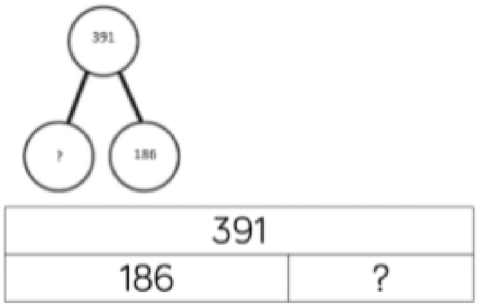
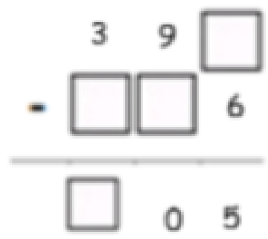
48-7</td>
<td>Children to represent the base 10 pictorially.


48-7</td>
<td>Column method or children could subtract 7.

$\begin{array}{crr} & 4 & 8 \\ - & & 7 \\ \hline & 4 & 1 \end{array}$</td>
</tr>
</table>

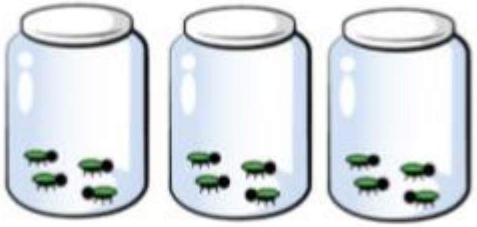
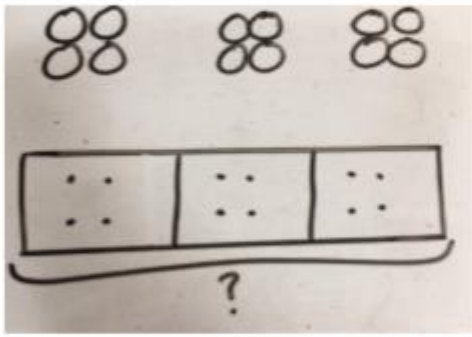
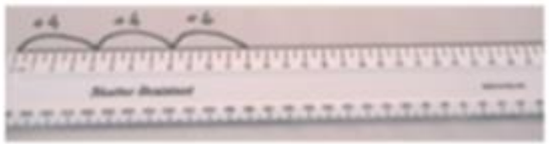
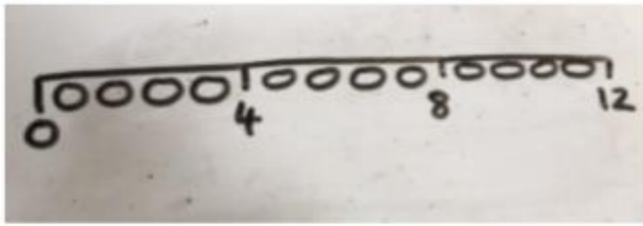
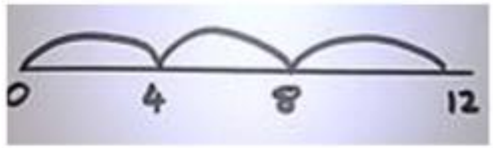
Column method using base 10 and having to exchange.  41–26	Represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Column method using place value counters.  234–88	Represent the place value counters pictorially; remembering to show what has been exchanged.  	Formal column method. Children must understand what has happened when they have crossed out digits. $$\begin{array}{r} 2\,\overset{2}{\cancel{3}}\,\overset{1}{4} \\ -\ \ 88 \\ \hline 6 \\ \hline \end{array}$$

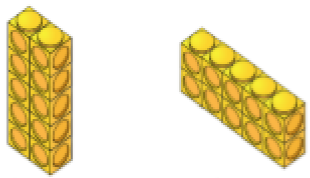
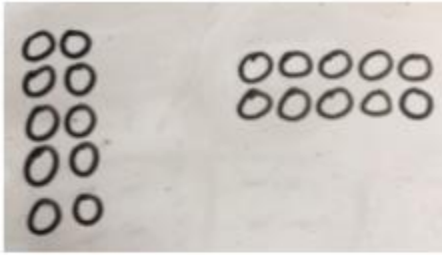
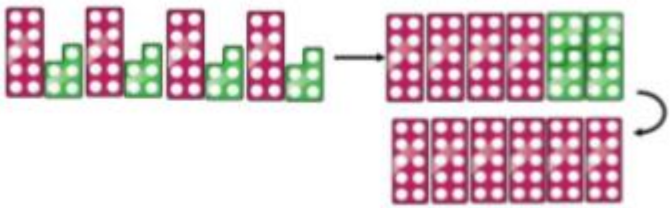
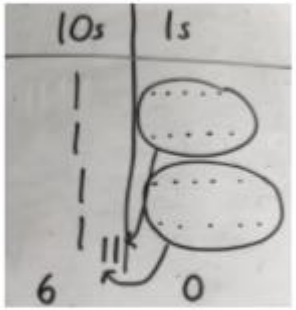
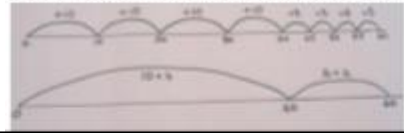
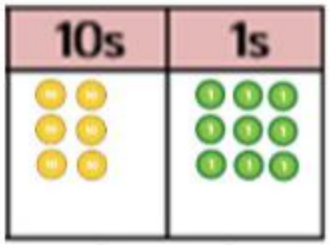
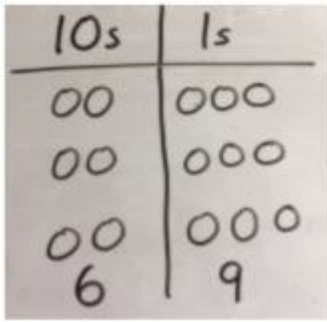
Conceptual variation; different ways to ask children to solve 391-186

 **Part whole**	**Word problems:** Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	 = 391 – 186 391 -186 What is 186 less than 391? **Formal written calculation**	Missing digit calculations  **Problem solving**

Calculation policy: Multiplication

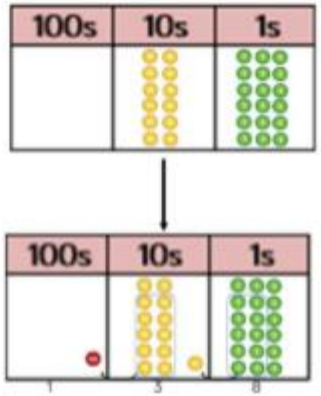
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.  	Children to represent the practical resources in a picture and use a bar model. 	4 + 4 + 4 = 12 3 × 4 = 12
Number lines to show repeated groups   Cuisenaire rods can be used too. 3 × 4	Represent this pictorially alongside a number line e.g.  3 × 4	Abstract number line showing three jumps of four.  3 × 4 = 12

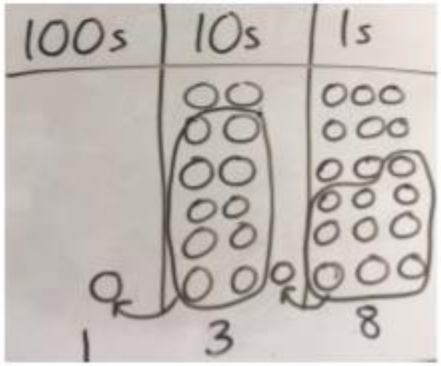
Use arrays to illustrate commutativity. Counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3 × 23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 → 20 3 $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times \ \ 3 \\ \hline 69 \\ \hline \end{array}$

Formal column method with place value	Children to represent the counters/base 10,	Formal written method

counters.
6 x 23



pictorially e.g. the image below.



6 x 23 =

$$\begin{array}{r} 23 \\ \times\ \ 6 \\ \hline 138 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

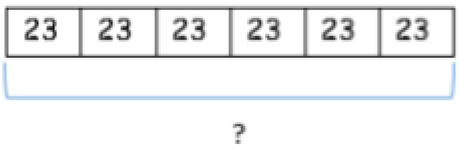
Expanded method When children start to multiply 3 digit × 3 digit and 4 digit × 2 digit etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124. To get 2480 they have solved 20 × 124.

$$\begin{array}{rrrrr} & & 1 & 2 & 4 \\ \times & & & 2 & 6 \\ \hline & & 7 & 4 & 4 \\ & 2 & 4 & 8 & 0 \\ \hline & 3 & 2 & 2 & 4 \\ \hline & {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 x 23



Bar model

Word problem. Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Formal written method

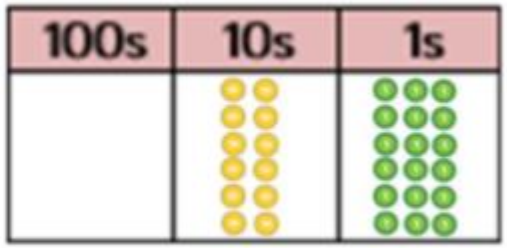
Find the product of 6 and 23

6 × 23 =

$\boxed{}$ = 6 × 23

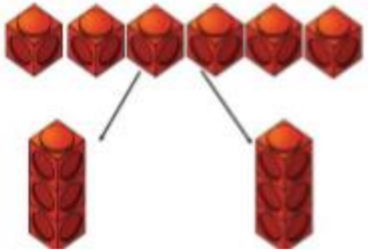
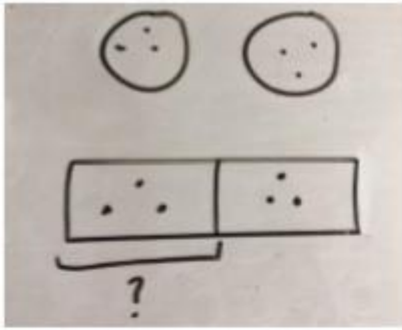
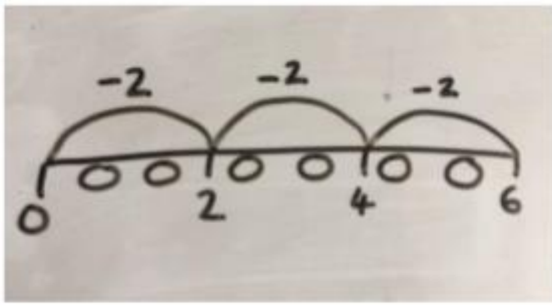
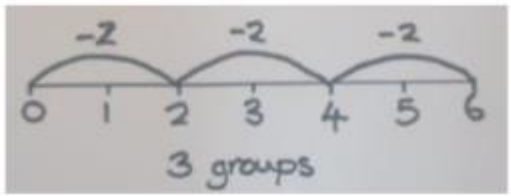
$$\begin{array}{r} 6 \\ \times\ \underline{23} \\ \underline{} \end{array} \qquad \begin{array}{r} 23 \\ \times\ \ \underline{6} \\ \underline{} \end{array}$$

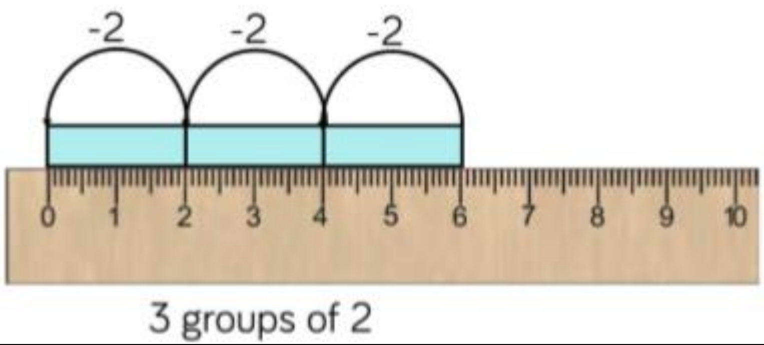
What is the calculation?
What is the product?

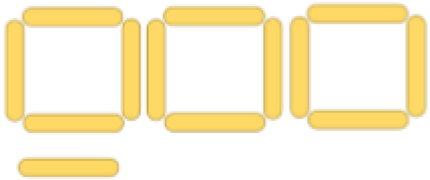
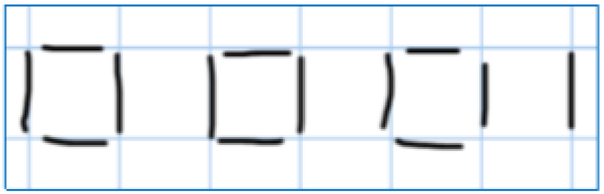
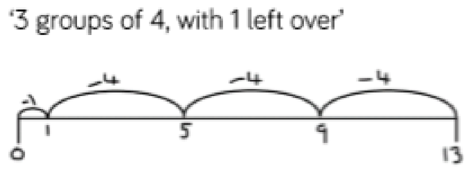


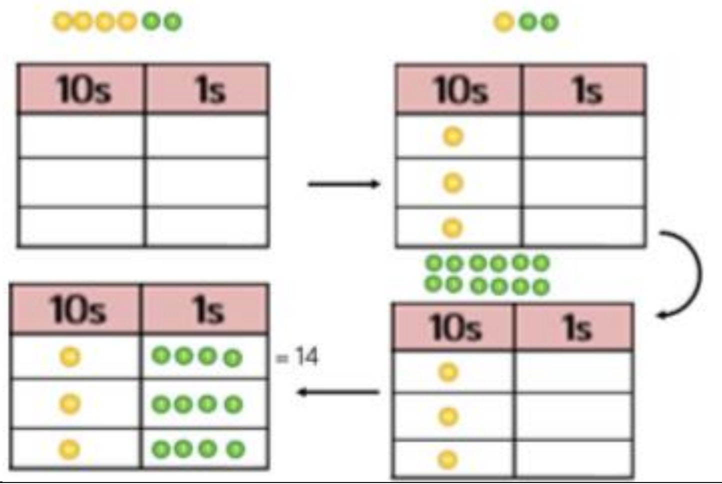
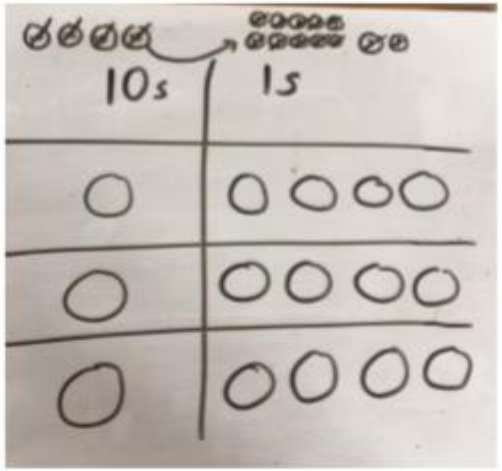
Calculation policy: Division

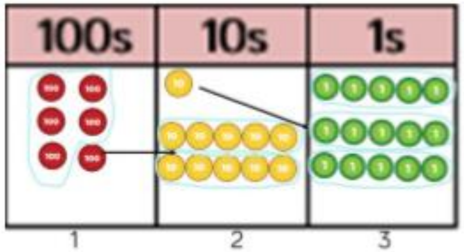
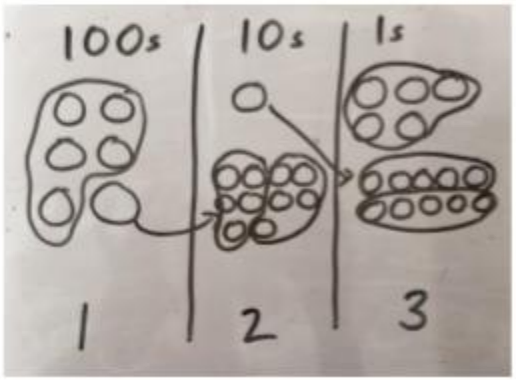
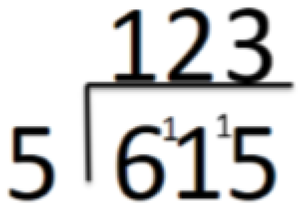
Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2  	Represent the sharing pictorially.  	Children should also be encouraged to use their 2 times table facts. 6 ÷ 2 = 3 | 3 | 3 |
Repeated subtraction using Cuisenaire rods above a ruler or number line. 6 ÷ 2	Children to represent repeated subtraction pictorially.  	Abstract number line to represent the equal groups that have been subtracted. 

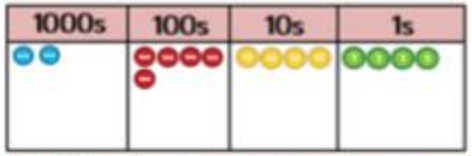
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14	Children to represent the place value counters pictorially.	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3

		42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

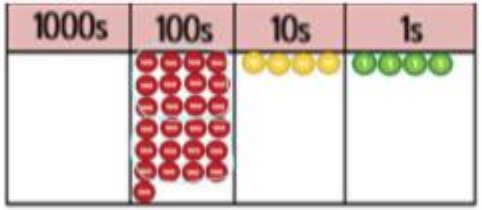
Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially. 	Children to the calculation using the short division scaffold. 

Long division using place value counters

$2544 \div 12$

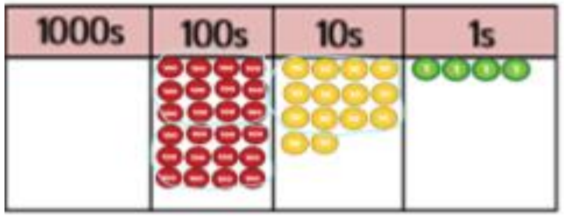


We can't group 2 thousands into groups of 12 so will exchange them.



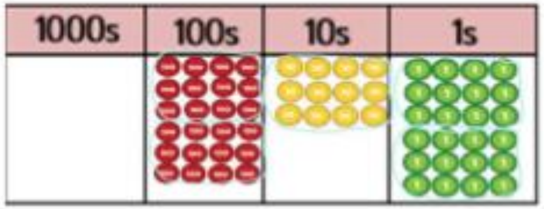
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

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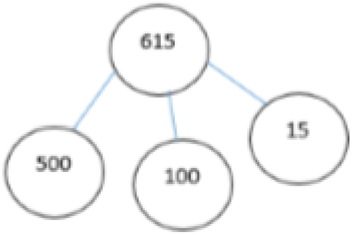
Conceptual variation; different ways to ask children to solve 615 ÷ 5

Part whole: Using the part whole model below, how can you divide 615 by 5 without using short division?  	Word problem: I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	Formal written method $5\overline{)615}$ $615 \div 5 =$ $\boxed{} = 615 \div 5$	What is the calculation? What is the answer?